Q/ Show that
$$\lim_{n\to\infty} (n)^{1/n} = 1$$

Sol^n^ $(n)^{1/n}$ can be written as

$$(n)^{1/n} = \left(1\cdot\frac{2}{1}\cdot\frac{3}{2}\cdot\frac{4}{3}\cdot\frac{5}{4}\cdot\frac{6}{5}\cdots\frac{n}{n-1}\right)^{1/n}$$

$\Rightarrow$ Since $\lim_{n\to\infty}\left(\frac{n}{n-1}\right) = 1$

$\Rightarrow$ $\lim_{n\to\infty} (n)^{1/n} = 1.$ H.P.

Sub sequence (उपानुक्रम)

Definition: let $\langle x_n \rangle$ be a sequence and $\langle n_k \rangle$ be the strictly increasing sequence i.e. $n_1 < n_2 < n_3 < \cdots$, then $\langle x_{n_k} \rangle$ is called a sub-sequence of $\langle x_n \rangle$.

or. we can say that a sequence which is formed by the elements of $\langle x_n \rangle$ by preserving the order which was followed for $\langle x_n \rangle$ is called a sequence.

Ex. let $\langle x_n \rangle$ be a sequence
$\Rightarrow \langle x_2, x_4, x_6, x_8, \cdots x_{2n} \cdots \rangle$ is a subsequence of $\langle x_n \rangle$ or $\langle x_{2n} \rangle$ is a subsequence of $\langle x_n \rangle$.

Q1 Prove that the following sequence is convergent:

$x_1 = \sqrt{3}$ and $x_{n+1} = \sqrt{3x_n}$

Sol^n: Given that $x_{n+1} = \sqrt{3x_n}$

$\Rightarrow x_2 = \sqrt{3x_1} = \sqrt{3\sqrt{3}}$

$\Rightarrow$ Terms of the sequence $\langle x_n \rangle$ are

$\cancel{\$}, \sqrt{3}, \sqrt{3\sqrt{3}}, \sqrt{3\sqrt{3\sqrt{3}}} \text{ ------ } \ldots$

$\Rightarrow x_2 > x_1 ; x_3 > x_2$ etc

$\Rightarrow x_{n+1} > x_n \Rightarrow \sqrt{3x_{n+1}} > \sqrt{3x_n}$

$\Rightarrow x_{n+2} > x_{n+1}$

$\Rightarrow$ By mathematical induction it is true for arbitrary value of n

$\Rightarrow a_{n+2} > a_{n+1}$

Again $x_1 < 3$, $x_2 < 3$, $x_3 = \sqrt{3x_2}$

$\Rightarrow x_3 < 3$ etc

$\Rightarrow \sqrt{3} < x_n < 3 \quad \forall n \in N$

Since $\lim\limits_{n\to\infty} x_{n+1} = \sqrt{3 \lim\limits_{n\to\infty} x_n}$

$\Rightarrow l = \sqrt{3l}$

$\Rightarrow l^2 - 3l = 0 \Rightarrow l = 0, 3$

l can't be zero as $x_1 = \sqrt{3}$

$\Rightarrow l = 3$

$\Rightarrow \langle x_n \rangle$ converges to 3.

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Th. Show that if a sequence $\langle x_n \rangle$ converges to l, then subsequence $\langle x_{n_k} \rangle$ also converges to l.

Proof :- Since $\lim_{n \to \infty} x_n = l$

$\Rightarrow$ for a given $\epsilon > 0$, $\exists\, n_0 \in N$ s.t.

$|x_n - l| < \epsilon \quad \forall\, n > n_0$ —— (1)

In the subsequence $\langle x_{n_k} \rangle$ n_k is an increasing form and for ~~~~ n_0 $\exists\, n_{k_0}$ st. $x_{n_{k_0}}$ is the element of sequence x_{n_k} st. $n_k > n_{k_0}$.

$\Rightarrow$ for subsequence $\langle x_{n_k} \rangle$ $\exists\, n_{k_0} \in \{n_k\}$ st. after n_{k_0}; $|x_{n_k} - l| < \epsilon$

$\Rightarrow$ $\langle x_{n_k} \rangle$ converges to l.

H.P.

Cauchy's sequence

Def: A sequence $\langle x_n \rangle$ is called a fundamental or Cauchy's sequence if for an arbitrary $\epsilon > 0$, $\exists$ a positive number n_0 st

$$|x_n - x_m| < \epsilon \quad \forall\, n \geq n_0,\ m \geq n_0.$$

or we can say that if $\langle x_n \rangle$ is a Cauchy's sequence, then for arbitrary $\epsilon > 0$, $\exists\, n_0 \in N$ st.

$$|x_{n+p} - x_n| < \epsilon \quad \forall\, n \geq n_0 \text{ and } p \in N.$$